UNIT 6

Combined Operations of Common Fractions and Mixed Numbers

Objectives *After studying this unit you should be able to*

- Solve problems that involve combined operations of fractions and mixed numbers.
- Solve complex fractions.

Before a part is machined, the sequence of machining operations, the machine setup, and the working dimensions needed to produce the part must be determined. In actual practice, calculations of machine setup and working dimensions require not only the individual operations of addition, subtraction, multiplication, and division, but a combination of two or more of these operations.

Order of Operations for Combined Operations

Procedure

- **Do all the work in the parentheses first.** Parentheses are used to group numbers. In a problem expressed in fractional form, the numerator and the denominator are each considered as being enclosed in parentheses. Brackets, [ ], and braces, { }, are used for "nesting" one group within another. They are treated the same as parentheses. On your calculator, use the parentheses, ( ), symbols.

Example 1 Compute $\{5 + [7 - 3(6 - 4) + 2] - 6\} + 1$.

Begin with the innermost grouping symbols, the parentheses.

Next, work within the brackets.

Finally, do the operations inside the braces.

$$\begin{aligned}
&\{5 + [7 - 3(6 - 4) + 2] - 6\} + 1 \\
&= \{5 + [7 - 3(2) + 2] - 6\} + 1 \\
&= \{5 + [7 - 6 + 2] - 6\} + 1 \\
&= \{5 + [3] - 6\} + 1 \\
&= \{5 + 3 - 6\} + 1 \\
&= \{2\} + 1 \\
&= 3 \quad \text{Ans}
\end{aligned}$$

$$\frac{4\frac{3}{4} - \frac{1}{2}}{10 + 6\frac{5}{8}} = \left(4\frac{3}{4} - \frac{1}{2}\right) \div \left(10 + 6\frac{5}{8}\right)$$

If an expression contains nested parentheses, do the work within the innermost parentheses first.

- **Do multiplication and division next.** Perform multiplication and division in order from left to right.
- **Do addition and subtraction last.** Perform addition and subtraction in order from left to right.

Example 2 Find the value of $\left(1\frac{2}{5}+\left(\frac{7}{3}-\frac{9}{5}\right)\right)+\frac{1}{4}$.

There are two sets of parentheses with one set nested inside the other.

Begin with the innermost parentheses: $\frac{7}{3}-\frac{9}{5}=\frac{8}{15}$

The result is $\left(1\frac{2}{5}+\frac{8}{15}\right)+\frac{1}{4}$.

Next, perform the operation in the remaining parentheses: $1\frac{2}{5}+\frac{8}{15}=1\frac{14}{15}$

The result is $1\frac{14}{15}+\frac{1}{4}$.

Add these two fractions: $1\frac{14}{15}+\frac{1}{4}=2\frac{11}{60}$ Ans

Combining Addition and Subtraction

Example 1 Find the value of $3\frac{1}{2}-\frac{3}{8}+\frac{5}{16}$.

There are no parentheses and there is no multiplication or division. So, perform addition and subtraction in order from left to right.

Subtract $\frac{3}{8}$ from $3\frac{1}{2}$. $3\frac{1}{2}-\frac{3}{8}=3\frac{1}{8}$

Add $3\frac{1}{8}$ to $\frac{5}{16}$. $3\frac{1}{8}+\frac{5}{16}=3\frac{7}{16}$ Ans

Example 2 Find x, the distance from the base of the plate in Figure 6-1 to the center of hole #2. All dimensions are in inches.

$$x=\frac{9}{16}"+2\frac{1}{8}"-\frac{13}{32}"$$

Add. $\frac{9}{16}"+2\frac{1}{8}"=2\frac{11}{16}"$

Subtract. $2\frac{11}{16}"-\frac{13}{32}"=2\frac{9}{32}"$ Ans

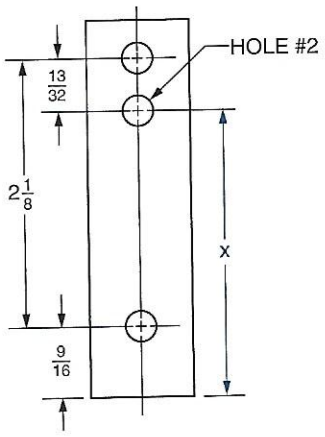


Figure 6-1

Combining Multiplication and Division

Example 1 Find the value of $\frac{2}{3}\times 8\div 2\frac{1}{2}$.

There are no parentheses. Perform multiplication and division in order from left to right.

Multiply. $\frac{2}{3}\times 8=\frac{2\times 8}{3\times 1}=\frac{16}{3}$

Divide. $\frac{16}{3}\div 2\frac{1}{2}=\frac{16}{3}\div\frac{5}{2}=\frac{16}{3}\times\frac{2}{5}=\frac{32}{15}=2\frac{2}{15}$ Ans

Example 2 The stainless-steel plate shown in Figure 6-2 has slots that are of uniform length and equally spaced within a distance of $33\frac{1}{2}$ inches. The time required to rough and

finish mill a one-inch length of slot is $\frac{7}{10}$ minute. How many minutes are required for the tool to cut all the slots? Disregard the time required to reposition the part. All dimensions are in inches.

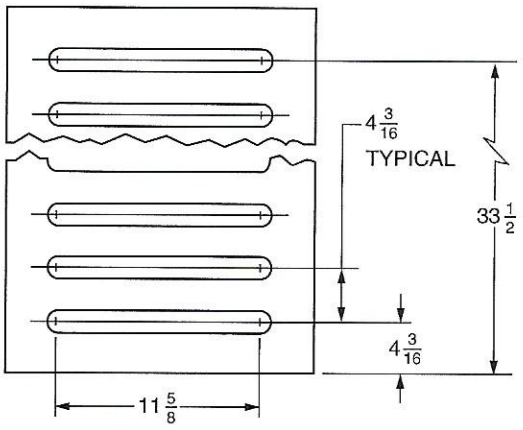


Figure 6-2

The number of grooves in $33\frac{1}{2}'' = 33\frac{1}{2} \div 4\frac{3}{16}$.

The time required to cut 1 groove $= \frac{7}{10} \times 11\frac{5}{8}$.

Total time equals the number of grooves multiplied by the time for each groove.

$$33\frac{1}{2} \div 4\frac{3}{16} \times \frac{7}{10} \times 11\frac{5}{8}$$

Divide. $33\frac{1}{2} \div 4\frac{3}{16} = \frac{67}{2} \times \frac{16}{67} = 8$

Multiply. $8 \times \frac{7}{10} \times 11\frac{5}{8} = \frac{8}{1} \times \frac{7}{10} \times \frac{93}{8} = 65\frac{1}{10}$

Total time $= 65\frac{1}{10}$ minutes Ans

Combining Addition, Subtraction, Multiplication, and Division

Example 1 Find the value of $7\frac{5}{6} + 5\frac{1}{2} \div \frac{3}{4} - 10 \times \frac{7}{16}$.

First divide and multiply.

$$7\frac{5}{6} + \underbrace{5\frac{1}{2} \div \frac{3}{4}} - \underbrace{10 \times \frac{7}{16}}$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$
$$7\frac{5}{6} + \underbrace{5\frac{1}{2} \times \frac{4}{3}} - \underbrace{\frac{10}{1} \times \frac{7}{16}}$$
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

Next add and subtract. $7\frac{5}{6} + \quad 7\frac{1}{3} \quad - \quad 4\frac{3}{8} = 10\frac{19}{24}$ Ans

Example 2 Find the value of $\left(7\frac{5}{6} + 5\frac{1}{2}\right) \div \frac{3}{4} - 10 \times \frac{7}{16}$.

First do the work in parentheses. $\left(7\frac{5}{6} + 5\frac{1}{2}\right) = 7\frac{5}{6} + 5\frac{3}{6} = 13\frac{1}{3}$

Next divide and multiply.

$$13\frac{1}{3} \div \frac{3}{4} = \frac{40}{3} \times \frac{4}{3} = \frac{160}{9} = 17\frac{7}{9}$$

$$10 \times \frac{7}{16} = 4\frac{3}{8}$$

Then add and subtract.

$$17\frac{7}{9} - 4\frac{3}{8} = 13\frac{29}{72} \quad \text{Ans}$$

➤ **Note:** This example is the same as the preceding example except for the parentheses.

Complex Fractions

A *complex fraction* is an expression in which either the numerator or denominator or both are fractions or mixed numbers. A fraction indicates a division operation. Therefore, complex fractions can be solved by dividing the numerator by the denominator.

$$\frac{\frac{5}{9}}{\frac{1}{3}} = \frac{5}{9} \div \frac{1}{3}$$

Example Find the value of $\dfrac{5\frac{7}{8} + 2\frac{3}{4}}{3\frac{15}{16} - 1\frac{1}{8}}$.

➤ **Note:** The complete numerator is divided by the complete denominator. Therefore, parentheses are used to indicate that addition in the numerator and subtraction in the denominator must be performed before division.

$$\frac{5\frac{7}{8} + 2\frac{3}{4}}{3\frac{15}{16} - 1\frac{1}{8}} = \left(5\frac{7}{8} + 2\frac{3}{4}\right) \div \left(3\frac{15}{16} - 1\frac{1}{8}\right)$$

$$= \frac{69}{8} \div \frac{45}{16}$$

$$= \frac{\overset{23}{\cancel{69}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{15}{\cancel{45}}} = 3\frac{1}{15} \quad \text{Ans}$$

Application

Order of Operations for Combined Operations

1. Solve the following examples of combined operations.

a. $\frac{1}{2} + \frac{3}{16} - \frac{1}{4}$ _____

b. $3\frac{7}{8} - 2\frac{3}{16} + \frac{3}{8}$ _____

c. $\frac{3}{10} + 8\frac{2}{5} - 3\frac{1}{25}$ _____

d. $27 - 2\frac{2}{3} + 4\frac{1}{6}$ _____

e. $32\frac{1}{8} + 2\frac{3}{16} \times \frac{3}{4}$ _____

f. $\frac{7}{9} \times \left(\frac{2}{3} + 3\frac{5}{6}\right)$ _____

g. $12 - 4\frac{1}{2} \div \frac{1}{2} + 2\frac{3}{4}$ _____

h. $\left(16 - 4\frac{1}{2}\right) \div \frac{1}{2} + 5\frac{3}{8}$ _____

i. $\left(16 - 4\frac{1}{2}\right) \div \left(\frac{1}{2} + 2\frac{1}{8}\right)$ _____

j. $15\frac{1}{4} \times 1\frac{1}{3} + 2\frac{2}{3} \div 4\frac{5}{6}$ _____

Complex Fractions

2. Find the value of the following complex fractions.

a. $\dfrac{\frac{3}{4}}{\frac{1}{2}}$ _____

b. $\dfrac{3\frac{7}{8}}{5}$ _____

c. $\dfrac{\frac{15}{16}}{2\frac{1}{8}}$ _____

d. $\dfrac{\frac{1}{3}+\frac{5}{6}}{3\frac{3}{4}}$ _____

e. $\dfrac{6\frac{3}{4}-2\frac{7}{8}}{3\frac{1}{2}+1\frac{1}{16}}$ _____

f. $\dfrac{10\frac{1}{2}\times\frac{1}{2}}{4\div 2\frac{1}{4}}$ _____

Related Problems

3. Refer to the shaft shown in Figure 6-3. Determine the missing dimensions in the table using the dimensions given. All dimensions are in inches.

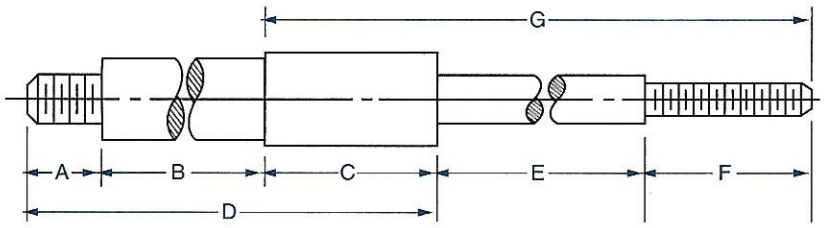


Figure 6-3

	A	B	C	D	E	F	G
a.	$\frac{1}{2}$		$1\frac{3}{8}$	$6\frac{3}{4}$		$\frac{15}{16}$	$7\frac{3}{8}$
b.		$3\frac{13}{32}$	$1\frac{5}{8}$	$5\frac{37}{64}$	$4\frac{3}{8}$	$\frac{3}{4}$	
c.	$\frac{7}{16}$	$4\frac{3}{32}$			$5\frac{1}{8}$	$\frac{27}{32}$	$7\frac{1}{32}$
d.	$\frac{5}{8}$		$1\frac{7}{16}$	$5\frac{31}{32}$		$\frac{7}{8}$	$7\frac{15}{16}$
e.		$3\frac{3}{4}$	$1\frac{11}{16}$	$6\frac{1}{32}$	$4\frac{61}{64}$	$\frac{25}{32}$	
f.	$\frac{11}{16}$	$4\frac{3}{16}$			$5\frac{3}{16}$	$\frac{7}{8}$	$7\frac{3}{64}$

4. The outside diameter of an aluminum tube is $3\frac{1}{16}$ inches. The wall thickness is $\frac{5}{32}$ inch. What is the inside diameter? _____

5. Four studs of the following lengths in inches are to be machined from bar stock: $1\frac{3}{4}$", $1\frac{7}{8}$", $2\frac{5}{16}$", and $1\frac{11}{32}$". Allow $\frac{1}{8}$ inch waste for each cut and $\frac{1}{32}$ inch on each end of each stud for facing. What is the shortest length of bar stock required so that only three cuts are needed? _____

6. Find dimensions A, B, C, and D of the idler bracket in Figure 6-4. All dimensions are in inches.

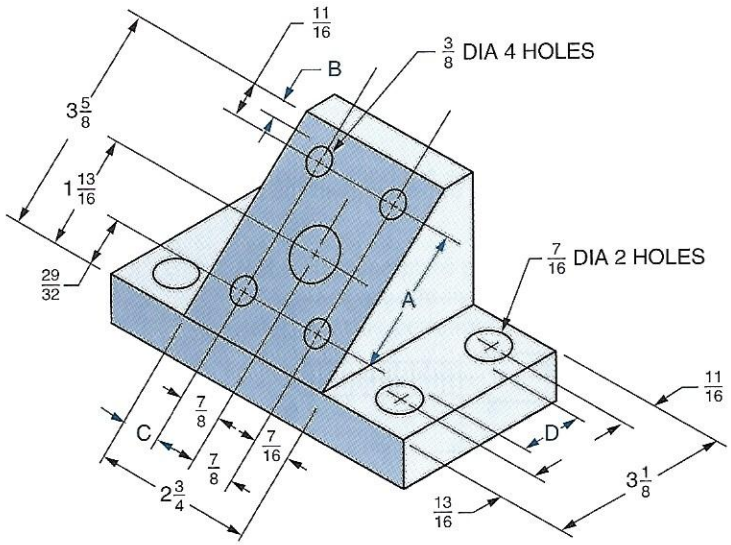


Figure 6-4

A = _____

B = _____

C = _____

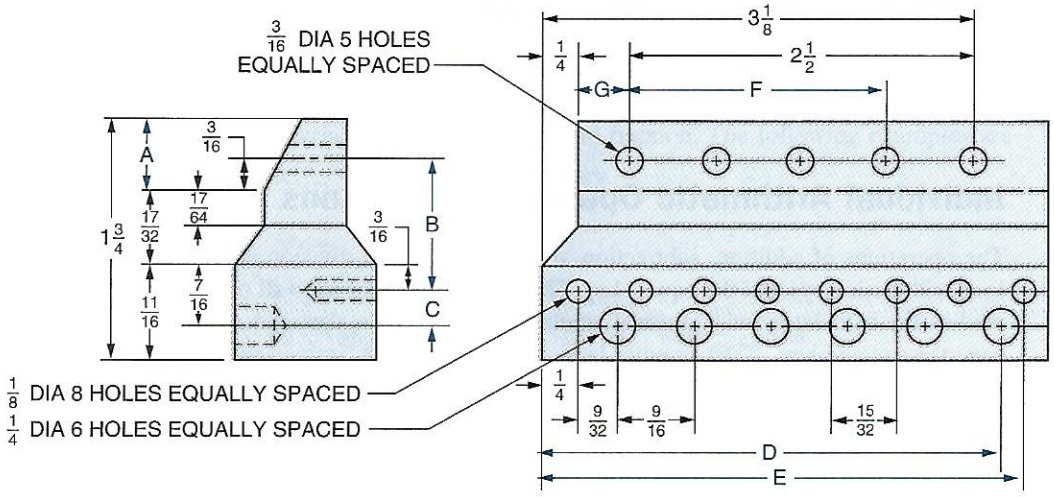
D = _____

7. How long does it take to cut a distance of $1\frac{1}{4}$ feet along a shaft that turns 150 revolutions per minute with a tool feed of $\frac{1}{32}$ inch per revolution? _____

8. An angle iron $47\frac{1}{2}$ inches long has two drilled holes that are equally spaced from the center of the piece. The center distance between the two holes is $19\frac{7}{8}$ inches. What is the distance from each end of the piece to the center of the closest hole? _____

9. A tube has an inside diameter of $\frac{3}{4}$ inch and a wall thickness of $\frac{1}{16}$ inch. The tube is to be fitted in a drilled hole in a block. What diameter hole should be drilled in the block to give $\frac{1}{64}$ inch total clearance? _____

10. Two views of a mounting block are shown in Figure 6-5. Determine dimensions A–G. All dimensions are in inches.



A = _____

B = _____

C = _____

D = _____

E = _____

F = _____

G = _____

Figure 6-5

11. The composition of an aluminum alloy by weight is $\frac{19}{20}$ aluminum and $\frac{1}{50}$ copper. The only other element in the alloy is magnesium. How many pounds of magnesium are required for casting 125 pounds of alloy? _____

12. Pieces of the following lengths are cut from a 15-inch steel bar: $2\frac{1}{2}$", $1\frac{3}{4}$", $1\frac{7}{8}$", and $\frac{5}{16}$". Allowing $\frac{1}{8}$ inch waste for each cut, what is the length of bar left after the pieces are cut? _____

UNIT 7

Computing with a Calculator: Fractions and Mixed Numbers

Objectives *After studying this unit you should be able to*

- Perform individual operations of addition, subtraction, multiplication, and division with fractions using a calculator.
- Perform combinations of operations with fractions using a calculator.

Fractions

Depending on the calculator, the fraction key (a b/c) (or Ab/c) is used when entering fractions and mixed numbers in a calculator. If your calculator has Ab/c, substitute it for all of the examples shown. The answers to expressions entered as fractions will be given as fractions or mixed numbers with the fraction in lowest terms.

Enter the numerator, press a b/c, and enter the denominator. The fraction is displayed with the symbol ⌟ or ⌜ between the numerator and denominator.